

Orange Public Schools

Office of Curriculum & Instruction
2019-2020 Mathematics Curriculum Guide



Second Grade

Unit 8:

Eureka - Module 6: Foundations Multiplication/ Division

June 8, 2020 – End of the School Year

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Yearlong Pacing Guide: Second Grade

<i>Eureka Math</i>	<i>Eureka Module Standards</i>
Unit 1- Module 1: Sum/ Difference to 20 Sept 9- Sept 27	2OA1, 2OA2, 2NBT5
Unit 2-Module 2: Add/ Subtract Length Sept 30- Oct 18	2MD1, 2MD2, 2MD3, 2MD4, 2MD5,2MD6
Unit 3- Module 3: Place Value, Count to 1000 Oct 21 – Nov 26	2NBT1, 2NBT2, 2NBT3, 2NBT4
Unit 4- Module 4: Add/ Subtract within 200 (Word Problems up to 100) Dec 2- Jan 31	2OA1, 2NBT5, 2NBT6, 2NBT7, 2NBT8, 2NBT9
Unit 5- Module 5: Add/ Subtract within 1000 (Word Problems up to 100) Feb 3- March 13	2NBT7, 2NBT8, 2NBT9
Unit 6- Module 7: Length, Money, Data March 16- May 8	2NBT5, 2MD1, 2MD2, 2MD3, 2MD4, 2MD5, 2MD6, 2MD8, 2MD9, 2MD10
Unit 7- Module 8: Time, Shapes, Fractions May 11- June 5	2MD7, 2G1, 2G3
Unit 8- Module 6: Foundations Multiplication Division June 8- EOSY	2OA3, 2OA4, 2G2

References

“Eureka Math” *Gt Minds*. 2018 < <https://greatminds.org/account/products>

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Module 6

Essential Questions

- Arrays are a way of representing both repeated addition and skip counting.
- Arrays should be identified in rows and then columns.

Enduring Understandings

- How is repeated addition related to skip counting.
- How are arrays and repeated addition related?
- How can we a model repeated addition equation with an array?

Performance Overview

- In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups, and progress to pictorial representations where they may begin by circling a group of 5 stars, adding 5 more, and then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition equation. Students calculate the repeated addition sums by adding on to the previous addends, step-by-step, or by grouping the addends into pairs and adding. By the end of Topic A, students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit. Hence, they begin their experience toward understanding that any unit may be counted. .This is the bridge between Grades 2 and 3. Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.
- In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time and express the total via repeated addition equations. As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically. .Then, they use same size square tiles, moving them closer together in preparation for composing rectangles
- In Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems. In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they need in preparation for Grade 3’s area content. They use same-size squares to tile a rectangle with no gaps or overlaps and then count to find the total number of squares that make up the rectangle.
- Topic D focuses on doubles and even numbers, thus setting the stage for the multiplication table of two in Grade 3

Module 6: Foundations of Multiplication and Division

Pacing:

June 8, 2020- End of School Year

Suggested Instructional Days: 15

Topic	Lesson	Student Lesson Objective/ Supportive Videos
Topic A: Formation of Equal Groups	Lesson 1&2	Use manipulatives to create equal groups. Use math drawings to represent equal groups, and relate to repeated addition https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 4	Represent equal groups with tape diagrams, and relate to repeated addition https://www.youtube.com/watch?v
Topic B: Arrays and Equal Groups	Lesson 5	Compose arrays from rows and columns and count to find the total using objects. https://www.youtube.com/watch?v
	Lesson 6	Decompose arrays into rows and columns, and relate to repeated Addition https://www.youtube.com/watch?v
	Lesson 7	Represent arrays and distinguish rows and columns using math Drawings https://www.youtube.com/watch?v
	Lesson 9	Solve word problems involving addition of equal groups in rows and columns https://www.youtube.com/watch?v

Mid-Module Assessment Task

<p>Topic C: Rectangular Arrays as a Foundation For Multiplication and Division</p>	Lesson 10&11	Use square tiles to compose a rectangle and relate to the array model. https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 12	Use math drawings to compose a rectangle with square tiles https://www.youtube.com/watch?v
	Lesson 13	Use square tiles to decompose a rectangle https://www.youtube.com/watch?v
	Lesson 14	Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares https://www.youtube.com/watch?v
	Lesson 15	Use math drawings to partition a rectangle with square tiles, and relate to repeated addition https://www.youtube.com/watch?v
	Lesson 16	Use grid paper to create designs to develop spatial structuring https://www.youtube.com/watch?v
<p>Topic D: The Mean- ing of Even and Odd Num- bers</p>	Lesson 17	Relate doubles to even numbers and write number sentences to express the sums. https://www.youtube.com/watch?v
	Lesson 18	Pair objects and skip-count to relate to even numbers https://www.youtube.com/watch?v
	Lesson 19	Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers https://www.youtube.com/watch?v
	Lesson 20	Use rectangular arrays to investigate odd and even numbers https://www.youtube.com/watch?v
End-Module Assessment Task		

Modifications	
Special Education/ 504:	English Language Learners:
<p>-Adhere to all modifications and health concerns stated in each IEP.</p> <p>-Give students a Menu, allowing students to pick assignments from different levels based on difficulty.</p> <p>-Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time</p> <p>-Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or in writing , such as Read-Draw-Write</p> <p>-Provide breaks between tasks, use positive reinforcement, use proximity</p> <p>-Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives</p> <p>-Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18)</p> <p>-Strategies for Students with 504 Plans</p>	<p>- Use manipulatives to promote conceptual understanding and enhance vocabulary usage</p> <p>- Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction</p> <p>- During i-Ready lessons, click on “Español” to hear specific words in Spanish</p> <p>- Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information</p> <p>- Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems</p> <p>- Utilize program translations (if available) for L1/ L2 students</p> <p>- Reword questions in simpler language</p> <p>- Make use of the ELL Mathematical Language Routines (click here for additional information)</p> <p>-Scaffolding instruction for ELL Learners</p> <p>-Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)</p>
Gifted and Talented:	Students at Risk for Failure:
<p>- Elevated contextual complexity</p> <p>- Inquiry based or open ended assignments and projects</p> <p>- More time to study concepts with greater depth</p> <p>- Promote the synthesis of concepts and making real world connections</p> <p>- Provide students with enrichment practice that are imbedded in the curriculum such as:</p> <ul style="list-style-type: none"> ● Application / Conceptual Development ● Are you ready for more? <p>- Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20)</p> <p>- Provide opportunities for math competitions</p> <p>- Alternative instruction pathways available</p>	<p>- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum</p> <p>- Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer Support, one on one instruction</p> <p>- Assure constant parental/ guardian contact throughout the year with successes/ challenges</p> <p>- Provide academic contracts to students and guardians</p> <p>- Create an interactive notebook with samples, key vocabulary words, student goals/ objectives.</p> <p>- Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessons.</p> <p>-Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)</p>

21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

<https://www.state.nj.us/education/cccs/2014/career/9.pdf>

- | | |
|--|--|
| <ul style="list-style-type: none">● CRP1. Act as a responsible and contributing citizen and employee.● CRP2. Apply appropriate academic and technical skills.● CRP3. Attend to personal health and financial well-being.● CRP4. Communicate clearly and effectively and with reason.● CRP5. Consider the environmental, social and economic impacts of decisions.● CRP6. Demonstrate creativity and innovation. | <ul style="list-style-type: none">● CRP7. Employ valid and reliable research strategies.● CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.● CRP9. Model integrity, ethical leadership and effective management.● CRP10. Plan education and career paths aligned to personal goals.● CRP11. Use technology to enhance productivity.● CRP12. Work productively in teams while using cultural global competence. |
|--|--|

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

Technology Standards:

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

<https://www.state.nj.us/education/cccs/2014/tech/>

8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

- A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
- B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
- C. **Communication and Collaboration:** Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
- D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
- E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
- F. **Critical thinking, problem solving, and decision making:** Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

- A. **The Nature of Technology: Creativity and Innovation-** Technology systems impact every aspect of the world in which we live.
- B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
- C. **Design:** The design process is a systematic approach to solving problems.
- D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
- E. **Computational Thinking: Programming-** Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections:	
English Language Arts:	
RF.2.4	Read with sufficient accuracy and fluency to support comprehension.
SL.2.1	Participate in collaborative conversations with diverse partners about <i>grade 2 topics and texts</i> with peers and adults in small and larger groups.
L.2.4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 2 reading and content, choosing flexibly from an array of strategies.

NJSL Unpacked Standards

2.OA.3

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Second graders apply their work with doubles to the concept of odd and even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends or doubles addition facts (e.g., $10 = 5 + 5$), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example: **Is 8 an even number? Justify your thinking.**

Student A

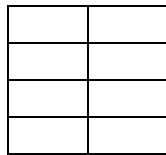
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.

Student B

I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

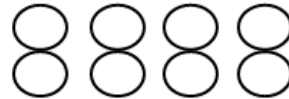
Student C

I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even.



Student D

I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number.



Student E

I know that 4 plus 4 equals 8. So 8 is an even number.

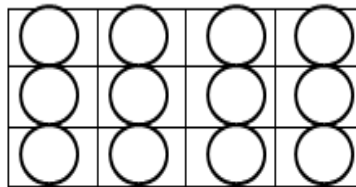
The focus of this standard is placed on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation and should not be used as the definition of an even number. (Van de Walle & Lovin, 2006, p. 292)

2.OA.4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends

Second graders use rectangular arrays to work with repeated addition, a building block for multiplication in third grade. A rectangular array is any arrangement of things in rows and columns, such as a rectangle of square tiles. Students explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

Example: What is the total number of circles below?



Student A

I see 3 counters in each column and there are 4 columns. So I added $3 + 3 + 3 + 3$. That equals 12.

$$3 + 3 + 3 + 3 = 12$$

Student B

I see 4 counters in each row and there are 3 rows. So I added $4 + 4 + 4$. That equals 12.

$$4 + 4 + 4 = 12$$

2.G.2

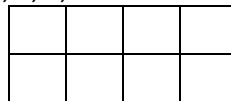
Partition a rectangle into rows and columns of same-size squares and count to find the total number of them

Second graders partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This work connects to the standard 2.OA.4 where students are arranging objects in an array of rows and columns. This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication.

Example:

Teacher: Partition the rectangle into 2 rows and 4 columns. How many small squares did you make?

Student: There are 8 squares in this rectangle. See- 2, 4, 6, 8. I folded the paper to make sure that they were all the same size.



M : Major Content

S: Supporting Content

A : Additional Content

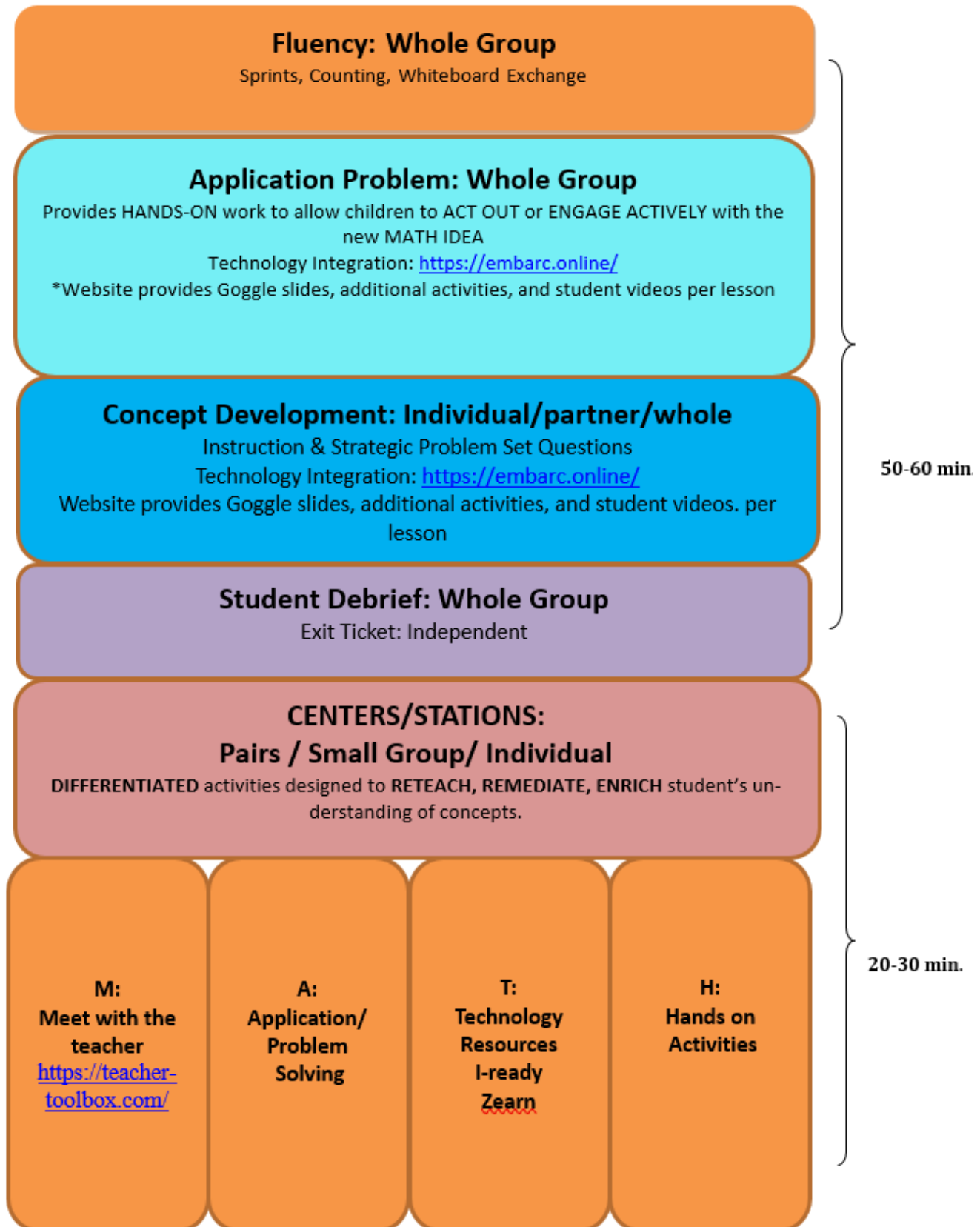
Module 6 Assessment Framework			
Assessment	NJSLS	Estimated Time	Format
Optional Mid-Module Assessment (Interview Style)	2.OA.4	1-2 blocks	Individual
Optional End-of- Module Assessment (Interview Style)	2.OA.3 2.OA.4 2.G.2	1 Block	Individual or Small Group with Teacher

Module 6 Performance Assessment/ PBL Framework			
Assessment	NJSLS	Estimated Time	Format
Module 6 Performance Task 1 <i>Playing Cards</i>	2.OA.3 2.OA.4	Up to 30 minutes	Individual or Small Group
Extended Constructed Response (ECR)* (click here for access)	Dependent on unit of study & month of administration	Up to 30 Minutes	Individual

Use the following links to access ECR protocol and district assessment scoring documents:

- [Assessment and Data in Mathematics Bulletin](#)
[ECR Protocol](#)

Second Grade Ideal Math Block



Lesson Structure:

Fluency:

- Sprints
- Whiteboard Exchange

Technology Integration:

Splat Sequences

[Which one doesn't belong?](#)

[Would you rather?](#)

Esti- Mysteries

Anchor Task:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Guided Practice/ Independent Practice : (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Technology Integration:

Think Central:

- Pre-Test
- Chapter Review
- Test Prep
- Performance Tasks

<https://embarc.online/>

[Virtual Manipulatives](#) for lessons

<http://nlvm.usu.edu/en/nav/vlibrary.html>

For videos that students can watch and interact with independently click [here](#)

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Centers:

- I-Ready: <https://login.i-ready.com/> *i-Ready* makes the promise of differentiated instruction a practical reality for teachers and students. It was designed to get students excited about learning and to support teachers in the challenge of meeting the needs of all learners. Through the power of one intuitive system whose pieces were built from the ground up to work together, teachers have the tools they need to ensure students are on the road to proficiency.
- Zearn: <https://www.zearn.org/> Zearn Math is a K-5 math curriculum based on Eureka Math with top-rated materials for teacher-led and digital instruction.
- Teacher Toolbox; <https://teacher-toolbox.com/> A digital collection of K-8 resources to help you differentiate instruction to students performing on, below, and above grade level.

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- It will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

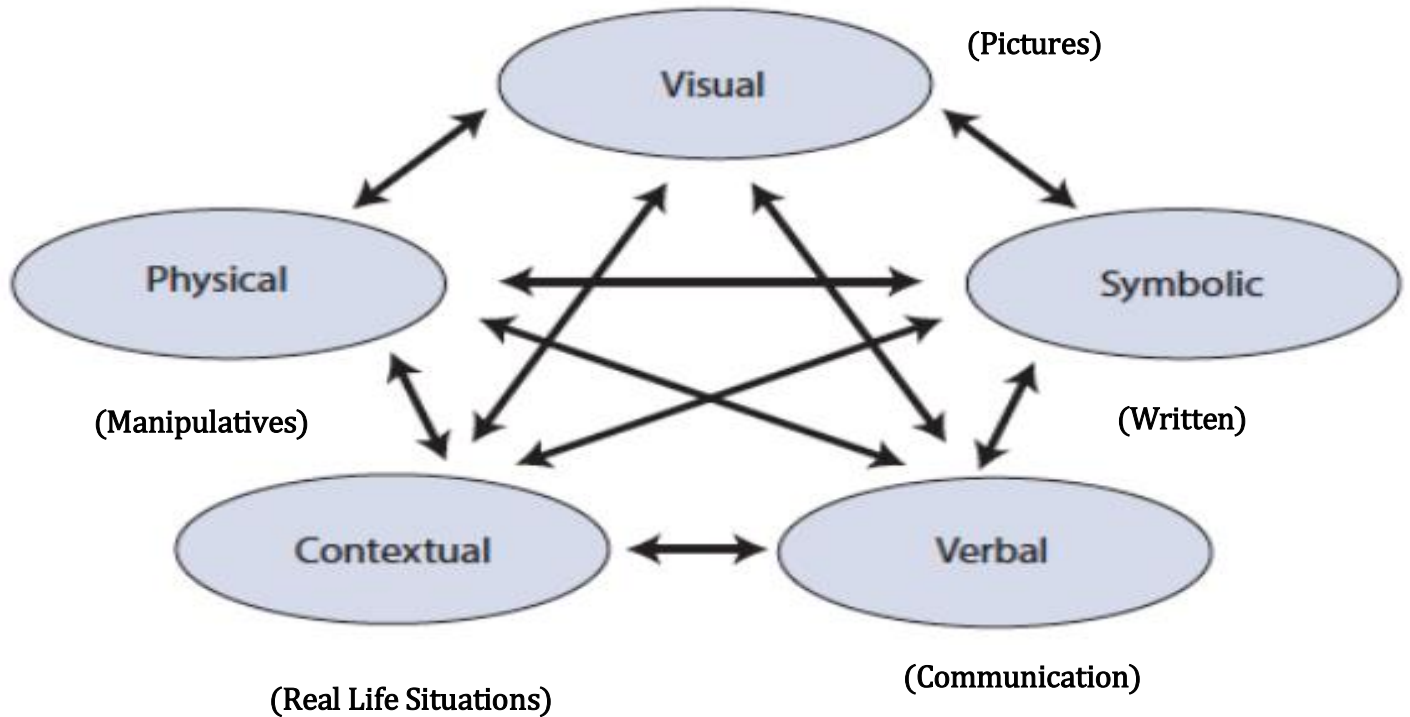
- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name: _____
 er: _____ Date: _____

Task: _____ School: _____ Teach-

"I CAN...."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



The most
important thing
is to NEVER
stop
questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

Dr.

100 questions that promote

Mathematical Discourse

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** ___?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is **mathematically correct**

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** _____?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



Help students learn to **conjecture, invent, and solve** problems

- 48 What would happen if ___?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram** or **make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



Help students learn to **connect mathematics, its ideas, and its application**

- 74 What is the **relationship** between ___ and ___?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?
- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to ___?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

Help students **persevere**

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?
- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

Help students **focus on the mathematics from activities**

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.OA.6 Add and Subtract within 10.

2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$);
- decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

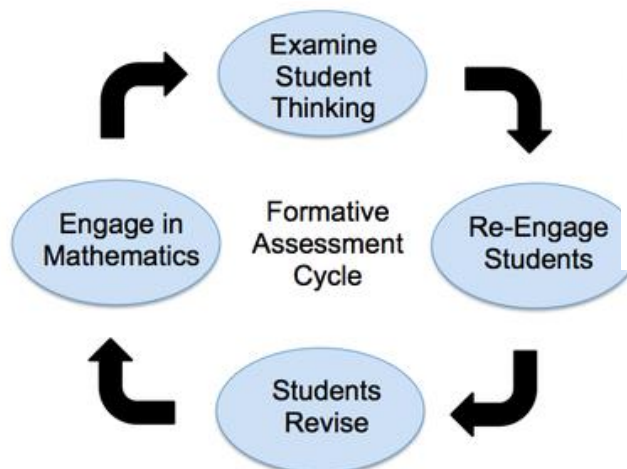
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(William 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Make sense of problems and persevere in solving them

1

In Kindergarten, students learn that doing math involves solving problems and discussing how they solved them. Students will begin to explain the meaning of a problem, and look for ways to solve it. Kindergarteners will learn how to use objects and pictures to help them understand and solve problems. They will begin to check their thinking when the teacher asks them how they got their answer, and if the answer makes sense. When working in small groups or with a partner they will listen to the strategies of the group and will try different approaches.

Reason abstractly and quantitatively

2

Mathematically proficient students in Kindergarten make sense of quantities and the relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Kindergarten, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, “There are 7 children on the playground and some children go line up. If there are 4 children still playing, how many children lined up?” Kindergarten students are expected to translate that situation into the equation: $7-4 = \underline{\quad}$, and then solve the task. Students also contextualize situations during the problem solving process. For example, while solving the task above, students refer to the context of the task to determine that they need to subtract 4 since the number of children on the playground is the total number of students except for the 4 that are still playing. Abstract reasoning also occurs when students measure and compare the lengths of objects.

3	<p>Construct viable arguments and critique the reasoning of others</p>
	<p>Mathematically proficient students in Kindergarten accurately use mathematical terms to construct arguments and engage in discussions about problem solving strategies. For example, while solving the task, “There are 8 books on the shelf. If you take some books off the shelf and there are now 3 left, how many books did you take off the shelf?” students will solve the task, and then be able to construct an accurate argument about why they subtracted 3 from 8 rather than adding 8 and 3. Further, Kindergarten students are expected to examine a variety of problem solving strategies and begin to recognize the reasonableness of them, as well as similarities and differences among them.</p>
4	<p>Model with mathematics</p>
	<p>Mathematically proficient students in Kindergarten model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Kindergarten students rely on concrete manipulatives and pictorial representations while solving tasks, but the expectation is that they will also write an equation to model problem situations. For example, while solving the task “there are 7 bananas on the counter. If you eat 3 bananas, how many are left?” Kindergarten students are expected to write the equation $7-3 = 4$. Likewise, Kindergarten students are expected to create an appropriate problem situation from an equation. For example, students are expected to orally tell a story problem for the equation $4+5 = 9$.</p>
5	<p>Use appropriate tools strategically</p>
	<p>Mathematically proficient students in Kindergarten have access to and use tools appropriately. These tools may include counters, place value (base ten) blocks, hundreds number boards, number lines, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students should also have experiences with educational technologies, such as calculators, virtual manipulatives, and mathematical games that support conceptual understanding. During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while solving the task “There are 4 dogs in the park. If 3 more dogs show up, how many dogs are they?” Kindergarten students are expected to explain why they used specific mathematical tools.”</p>
6	<p>Attend to precision</p>
	<p>Mathematically proficient students in Kindergarten are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Kindergarten describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.</p>

7	<p>Look for and make use of structure</p> <p>Mathematically proficient students in Kindergarten carefully look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property, in that $1+4 = 5$, and $4+1 = 5$. While decomposing teen numbers, students realize that every number between 11 and 19, can be decomposed into 10 and some leftovers, such as $12 = 10+2$, $13 = 10+3$, etc. Further, Kindergarten students make use of structures of mathematical tasks when they begin to work with subtraction as missing addend problems, such as $5 - 1 = \underline{\quad}$ can be written as $1 + \underline{\quad} = 5$ and can be thought of as how much more do I need to add to 1 to get to 5?</p>
8	<p>Look for and express regularity in repeated reasoning</p> <p>Mathematically proficient students in Kindergarten begin to look for regularity in problem structures when solving mathematical tasks. Likewise, students begin composing and decomposing numbers in different ways. For example, in the task “There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?” Kindergarten students are expected to realize that the 8 crayons could include 4 of each color ($4+4 = 8$), 5 of one color and 3 of another ($5+3 = 8$), etc. For each solution, students repeated engage in the process of finding two numbers that can be joined to equal 8.</p>

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

<u>5 Practices for Orchestrating Productive Mathematics Discussions</u>	
Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: _____

Time:

NJSLS:

Objective(s): By the end of this task, I will be able to:

- _____
- _____
- _____

Task(s):

- _____
- _____
- _____
- _____

Exit Ticket:

- _____
- _____
- _____

MATH WORKSTATION SCHEDULE

Week of: _____

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OB- SERVATIONAL DA- TA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	

Second Grade PLD Rubric

Got It		Not There Yet		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	PLD Level 4: 89% Strong Command	PLD Level 3: 79% Moderate Command	PLD Level 2: 69% Partial Command	PLD Level 1: 59% Little Command
<p>Student work shows distinct levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an efficient and logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows strong levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows moderate levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical but incomplete progression of mathematical reasoning and understanding. Contains minor errors.</p>	<p>Student work shows partial understanding of the mathematics.</p> <p>Student constructs and communicates an incomplete response based on student’s attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an incomplete or illogical progression of mathematical reasoning and understanding.</p>	<p>Student work shows little understanding of the mathematics.</p> <p>Student attempts to construct and communicates a response using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes limited evidence of the progression of mathematical reasoning and understanding.</p>
5 points	4 points	3 points	2 points	1 point

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

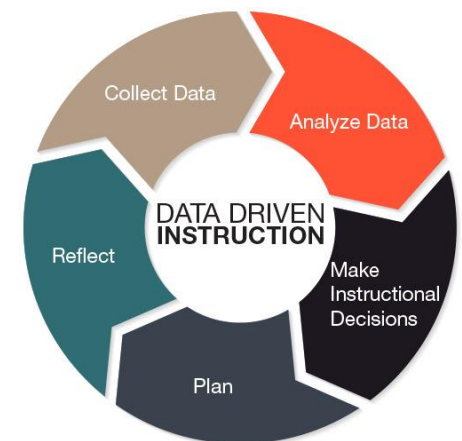
Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Data Analysis Form

School: _____ **Teacher:** _____ **Date:** _____

Assessment: _____ **NJSLS:** _____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the NJSL. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be “practice forward” (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are “practice forward” and denoted as “Individual”, “Partner/Group”, and “Individual w/Opportunity for Student Interviews¹”.
- Each Student Assessment Portfolio should contain a “Task Log” that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity “as a new and separate score” in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)².
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

2nd Grade Authentic Assessment #1 – Playing Games

A game company wants to create a new game. They are trying to figure out some of the rules of the game.

Part A

They want to use a spinner to distribute red and blue coins equally. If the spinner lands on an even number, the player gets a red coin. If the spinner lands on an odd number, the player gets a blue coin. If the game uses the spinner shown, how many numbers will result in a red coin? How many will result in a blue coin? Explain.



Part B

Will the spinner in Part A give the same number of red and blue coins? Explain.

Part C

Suppose gray coins are worth 3 points and black coins are worth 2 points. Write and solve an equation that shows how many points the gray coins are worth. Write and solve another equation that shows how many points the black coins are worth. Then write how many total points the player has.



2.OA.B.2

Fluently add and subtract within 20 using mental strategies.² By end of Grade 2, know from memory all sums of two one-digit numbers.

2.OA.C.3

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Part A

Part A	
Score	Description
2	<p>Student response includes the following 2 elements</p> <ul style="list-style-type: none"> • Number of Red/Blue Coins: 1 point • Explanation: 1 point <p><u>Sample Student Response:</u> 2 numbers will result in an even number and a red coin: 2, 4 3 numbers will result in an odd number and a blue coin: 1, 3, 5</p>
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Part B

Part B	
Score	Description
2	<p>Student response includes the following 2 elements</p> <ul style="list-style-type: none"> • Yes/No: 1 point • Explanation: 1 point <p><u>Sample Student Response:</u> No, since there are 3 options for blue coins and 2 options for red coins, there will be more options for blue coins to be given than red coins.</p>
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Part C

Score	Description
3	<p>Student response includes the following 3 elements</p> <ul style="list-style-type: none"> • Gray Coins: 1 point • Black Coins: 1 point • Total: 1 point <p><u>Sample Student Response:</u> Gray: $3 + 3 = 6$ points Black: $2 + 2 + 2 + 2 = 8$ points Total: $6 + 8 = 14$ points</p>
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using :</p> <ul style="list-style-type: none"> • Concepts of reading and writing numbers in the base ten number system • Strategies based on place value <p>Response includes an efficient and logical progression of steps.</p>	<p>Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Concepts of reading and writing numbers in the base ten number system • Strategies based on place value <p>Response includes a logical progression of steps</p>	<p>Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Concepts of reading and writing numbers in the base ten number system • Strategies based on place value <p>Response includes a logical but incomplete progression of steps. Minor calculation errors.</p>	<p>Student answers, clearly constructs, and communicates a complete response with major calculation errors and/or conceptual errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Concepts of reading and writing numbers in the base ten number system • Strategies based on place value <p>Response includes an incomplete or illogical progression of steps.</p>	<p>The student shows no work or justification.</p>

Core Instructional and Supplemental Materials (K-5)

EUREKA MATH V. 2019
(GREAT MINDS)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K (v. 2019.)	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
1	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
2	<ul style="list-style-type: none"> • Teacher Edition: Module 1-8 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-8 • Succeed Workbook Set: Module 1-8 • Practice Workbook, Fluency: Module 1-8
3	<ul style="list-style-type: none"> • Teacher Edition: Module 1-7 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
4	<ul style="list-style-type: none"> • Teacher Edition: Module 1-7 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
5	<ul style="list-style-type: none"> • Teacher Edition: Module 1-7 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
5	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6

MATH IN FOCUS v. 2015
(HOUGHTON MIFFLIN HARCOURT)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K	<ul style="list-style-type: none">• Teacher Edition (A & B)• Implementation Guide• Assessment Package• Enrichment Bundle• Extra Practice Set• Teacher and Student Activity Cards• Home -to- School Connection Book• Online Teacher Technology Kit• Big Book Set• Online Interactive Whiteboard Lessons	<ul style="list-style-type: none">• Student Edition A – Pt. 1• Student Edition A – Pt. 2• Student Edition B – Pt. 1• Student Edition B – Pt. 2• Online Student Technology Kit
1	<ul style="list-style-type: none">• Teacher Edition (A & B)• Implementation Guide• Assessment Package• Enrichment Bundle• Extra Practice Guide• Reteaching Guide• Home -to- School Connection Book• Online Teacher Technology Kit• Fact Fluency• Online Interactive Whiteboard Lessons	<ul style="list-style-type: none">• Student Texts (A & B)• Student Workbooks• Online Student Technology Kit• Student Interactivities
2-5	<ul style="list-style-type: none">• Teacher Edition (A & B)• Implementation Guide• Assessment Package• Enrichment Bundle• Extra Practice Guide• Transition Guides• Reteaching Guide• Home -to- School Connection Book• Online Teacher Technology Kit• Fact Fluency• Online Interactive Whiteboard Lessons	<ul style="list-style-type: none">• Student Texts (A & B)• Student Workbooks• Online Student Technology Kit• Student Interactivities

Supplemental Resources

Great Minds

<https://greatminds.org/>

Embarc

<https://embarc.online/>

Engage NY

[http://www.engageny.org/video-library?f\[0\]=im_field_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

Common Core Tools

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

Achieve the Core

<http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12>

Manipulatives

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

Illustrative Math Project : <http://illustrativemathematics.org/standards/k8>

Inside Mathematics: <http://www.insidemathematics.org/index.php/tools-for-teachers>

Sample Balance Math Tasks: <http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

Georgia Department of Education: <https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

Gates Foundations Tasks: <http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf>